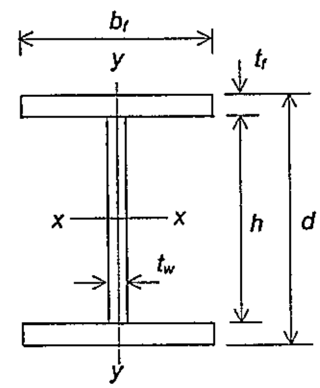
7.15 FÓRMULAS PARA DETERMINAR PROPIEDADES DE SECCIONES

PERFIL H SOLDADO



Área

$A = 2b_f t_f + ht_w$

Momento de inercia

$I_x = (b_f d^3 - (b_f - t_w)h^3)/12$

$I_y = (2t_f b_f^3 + ht_w^3)/12$

Módulo plástico

$Z_x = b_f t_f (h + t_f) + t_w h^2/4$

$Z_y = t_f b_f^2/2 + ht_w^2/4$

Propiedades flexo - torsionales

$J = (2b_f t_f^3 + (h + t_f)t_w^3)/3$

$C_w = t_f b_f^3(h + t_f)^2/24$

$$i_a = \sqrt{\frac{dI_y}{2S_x}}$$

$i_t = b_f t_f / d$

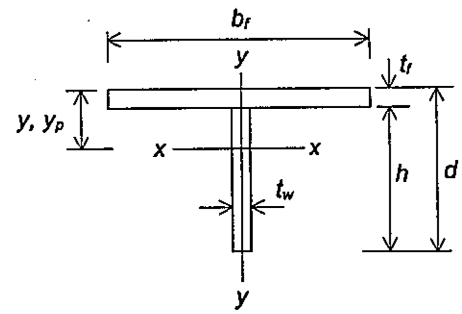
$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}}$$

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2$$

E = módulo de elasticidad del acero (200000 MPa)

G = módulo de corte del acero (77200 MPa)

PERFIL T SOLDADO



Área

$A = b_f t_f + h t_w$

Centro de gravedad

$y = t_f/2 + h d t_w/(2A)$

Centro plástico

$y_p \geq t_f$ si $A/2 \geq b_f t_f$

$y_p = (A/2 - b_f t_f)/t_w + t_f \quad ; \quad y_p \geq t_f$

$y_p = A/(2b_f) \quad ; \quad y_p < t_f$

Momento de inercia

$I_x = b_f t_f^3/12 + b_f t_f (y - t_f/2)^2 + t_w h^3/12 + t_w h(h/2 + t_f - y)^2$

$I_y = (t_f b_f^3 + h t_w^3)/12$

Módulo plástico

$Z_x = b_f t_f (y_p - t_f/2) + t_w (y_p - t_f)^2/2 + t_w (d - y_p)^2/2 \quad ; \quad y_p \geq t_f$

$Z_x = b_f (y_p^2 + t_f^2/2 - y_p t_f) + h t_w (h/2 + t_f - y_p) \quad ; \quad y_p < t_f$

$Z_y = (t_f b_f^2 + h t_w^2)/4$

Propiedades flexo - torsionales

$J = (b_f t_f^3 + (h + t_f/2) t_w^3)/3$

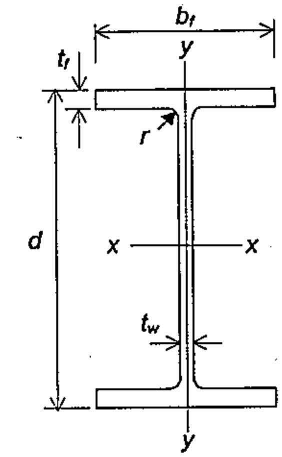
$C_w = (t_f^3 b_f^3/4 + t_w^3 (h + t_f/2)^3)/36$

$j = [\, \{(d - y)^4 - (y - t_f/2)^4\} t_w/4 - b_f t_f (y - t_f/2)\{(y - t_f/2)^2 + b_f^2/12\} \,] / (2I_x) + (y - t_f/2)$

$\bar{r}_o = ((y - t_f/2)^2 + (I_x + I_y)/A)^{1/2}$

$H = \beta = 1 - ((y - t_f/2) / \bar{r}_o)^2$

PERFIL H LAMINADO



Área

$A= 2b_f t_f + t_w (d - 2t_f) + (2r)^2 - \pi r^2$

Momento de inercia

$I_x = (b_f d^3 - (b_f - t_w)(d - 2t_f)^3)/12 + 0.8584r^2(d/2 - t_f - 0.2234r)^2 + 0.0302r^4$

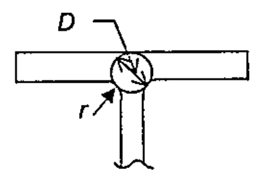
$I_y = t_f b_f^3/6 + (d - 2t_f)t_w^3 + 0.8584r^2(t_w/2 + 0.2234r)^2 + 0.0302r^4$

Módulo plástico

$Z_x = b_f t_f (d - t_f) + t_w (d/2 - t_f)^2 + 0.8584r^2(d/2 - t_f - 0.2234r)$

$Z_y = t_f b_f^2/2 + (d - 2t_f)t_w^2/4 + 0.8584r^2(t_w/2 + 0.2234r)$

Propiedades flexo - torsionales



$D = (t_f^2 + t_w^2/4 + 0.2929r(t_w + 2t_f)+0.1716r^2)/(t_f + 0.2929r)$

$\alpha = (0.15 + 0.10r / t_f)\, t_w / t_f$

$J = 2b_f t_f^3[\,1/3 - 0.21t_f\{1 - t_f^4/(12b_f^4)\}/b_f\,] + (d - 2t_f)t_w^3/3 + 2\alpha D^4$

$C_w = I_y (d - t_f)^2/4$

$$i_a = \sqrt{\frac{dI_y}{2S_x}}$$

$i_t = b_f t_f / d$

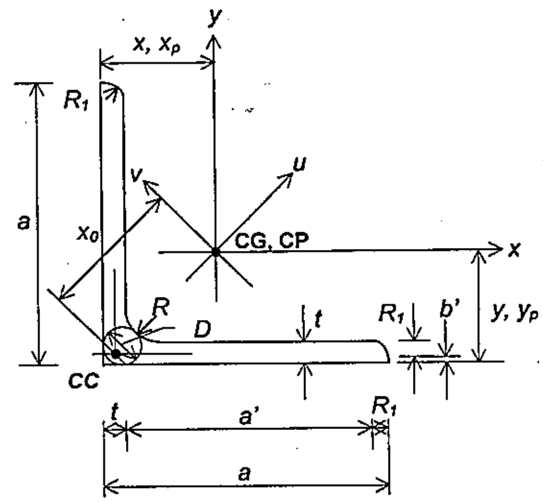
$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}}$$

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2$$

E = módulo de elasticidad del acero (200000 MPa)

G = módulo de corte del acero (77200 MPa)

PERFIL L LAMINADO



Parámetros auxiliares

$a' = a - t - R_1$

$b' = t - R_1$

Área

$A = t(2a - t) + 0.2146(R^2 - 2R_1^2)$

Centro de gravedad

$$x = y = \{ 6t(a(a + t) - t^2) + R_1^2(1.1504R_1 - 2.5752(a + t)) +$$

$$+ R^2(2.5752t + 0.5752R) \}/(12A)$$

Centro plástico

si $R \le 2.1587\, t$, $x_p = y_p \le t$

la ecuación trascendental que permite determinar x_p ($=y_p$) por algún método iterativo, es la siguiente :

$$(x_p - b')\sqrt{R_1^2 - (x_p - b')^2} + 2x_p(t + a') - R_1^2 Arccos\left(\frac{x_p - b'}{R_1}\right) - t^2 - 0.2146R^2 - 2ta' = 0$$

Momento de inercia

$$I_x = I_y = (a't^3 + a'^3t + t^4 + R_1b'^3 + R_1^3b')/12 + t(y - t/2)^2(a' + t) + 0.0075R^4 + R_1b'\{ (y - b'/2)^2 + (a' + t + R_1/2 - y)^2 \} +$$

$$+ 0.7854R_1^2 \{ (y - b' - 0.4244R_1)^2 + (a' + t + 0.4244R_1 - y)^2 \} + 0.2146R^2(y - t - 0.2234R)^2 + a't(y - t - a'/2)^2$$

$$I_{xy} = t(t/2 - x)(a^2 - 2ax + tx - t^2/2) - 0.1065(R^4/24 - R_1^4/12) + 0.2146R^2(x - t - 0.2234R)^2 -$$

$$- 0.4292R_1^2(a - x - 0.2234R_1)(t - x - 0.2234R_1)$$

$$I_u = I_x - I_{xy}$$

$$I_v = I_x + I_{xy}$$

Módulo plástico

$$Z_x = Z_y = a(t - x_p)^2 + t(a^2 - t^2 + 2tx_p - at)/2 + R^2 \{ 2.5752(t - x_p) + 0.5752R \}/12 - 0.2146R_1^2(a - t)$$

Propiedades flexo - torsionales

$$D = 0.8284t + 0.2426R$$

$$\alpha = 0.07 + 0.076R/t$$

$$J = at^3\left(\frac{1}{3} - 0.21\frac{t}{a}\left(1 - \frac{t^4}{12a^4}\right)\right) + t^3(a - t)\left(\frac{1}{3} - 0.105\frac{t}{a-t}\left(1 - \frac{t^4}{192(a-t)^4}\right)\right) + \alpha D^4$$

$$C_w = t^3(a - t/2)^3/18$$

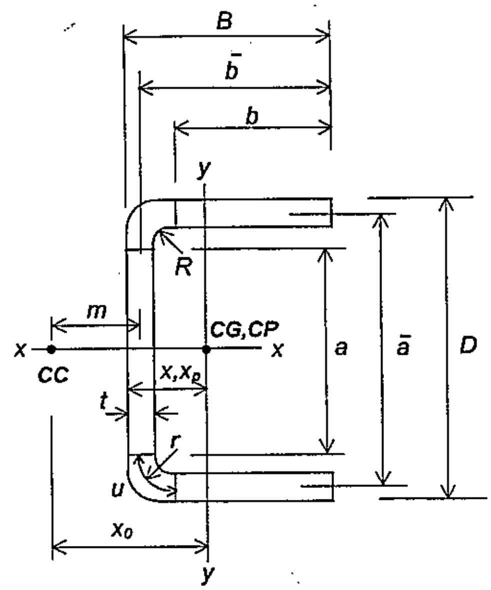
$x_0 = (x - t/2)\sqrt{2}$ (distancia entre el centro de gravedad CG y el centro de corte CC)

$$j = \sqrt{2}\, t(a - t/2)^4/(48I_v) + x_0$$

$$\bar{r}_0 = (x_0^2 + 2I_x/A)^{1/2}$$

$$H = \beta = 1 - (x_0/\bar{r}_0)^2$$

PERFIL C PLEGADO



Parámetros auxiliares

$r = R + t/2$
$u = \pi r/2$
$a = D - 2(t + R)$
$\bar{a} = D - t$
$b = B - t - R$
$\bar{b} = B - t/2$

Área

$A = t(a + 2(b + u))$

Centro de gravedad

$x = t(at/2 + b^2 + (b + u)(2r + t) - 2r^2 - t^2/6)/A$

Centro plástico

se distinguen 3 casos :

caso 1 : si $bt \geq A/4 \rightarrow x_p \geq (R + t)$ (eje en el tramo recto del ala)

$x_p = b/2 + 0.2146r + t/2 - a/4$

caso 2 : si $A_1 < (A/4 - at/2) \rightarrow t \leq x_p < (R + t)$ (eje en el codo) ; $A_1 = (r + t/2)^2 Arctan(t/r) - Rr/2$

$\theta_1 = (A/2 - at)/(2rt)$

$x_p = t/2 + r(1 - cos\theta_1)$

caso 3 : $x_p < t$ (eje en el alma)

en este caso, la ecuación trascendental que permite determinar x_p por algún método iterativo, es la siguiente :

$$\theta_2 = Arctan\left(\frac{\sqrt{2x_p(r+t/2)-x_p^2}}{r+t/2-x_p}\right)$$

$$ax_p + (r + t/2)^2(\theta_2 - \frac{1}{2} sen2\theta_2) - A/2 = 0$$

Momento de inercia

$I_x = 2t(0.0417a^3 + b(a/2 + r)^2 + u(a/2 + 0.637r)^2 + 0.149r^3)$

$I_y = 2t(0.0833b^3 + b(b/2 + r)^2 + 0.356r^3) - A(x - t/2)^2$

Módulo plástico

$Z_x = t(a^2/4 + \bar{a}b + \pi ra/2 + 2r^2 + t^2/6)$

Caso 1 : $x_p \geq (R + t)$

$Z_y = t(a(x_p - t/2) + \pi r(x_p - r - t/2) + (b + r + t/2 - x_p)^2 + (x_p - r - t/2)^2 + 2r^2 + t^2/6)$

Caso 2 : $t \leq x_p < (R + t)$

$Z_y = t\,[\,x_p\,(a + r(3\theta_1 - \pi/2) - 2b) + r^2(2sen\theta_1 - 3\pi\,\theta_1/2) + rt(\pi/2 - 3\theta_1)/2 + b(2B - b) - at/2\,]$

Caso 3 : $x_p < t$

$$Z_y = a(x_p^2 - tx_p + t^2/2) + \tfrac{3}{8}x_p(r + t/2)^2(\theta_2 - \tfrac{1}{2}\,\mathrm{sen}2\theta_2) + \tfrac{1}{2}\{\pi rt - (r + t/2)^2(\theta_2 - \tfrac{1}{2}\,\mathrm{sen}2\theta_2)\}(r + t/2 - x_p) + 2bt(B - b/2 - x_p)$$

Propiedades flexo - torsionales

$$m = 3\bar{b}^2/(\bar{a} + 6\bar{b})$$

$$J = t^3(a + 2b + 2u)/3$$

$$C_w = \frac{t\bar{a}^2\bar{b}^2}{12}\,\frac{\left(2\bar{a}^3\bar{b} + 3\bar{a}^2\bar{b}^2\right)}{\left(6\bar{a}^2\bar{b} + \bar{a}^3\right)}$$

$x_0 = x + m - t/2$ (distancia entre el centro de gravedad CG y el centro de corte CC)

$$\beta_w = -(t\,\bar{a}^3(x - t/2)/12 + t\,\bar{a}(x - t/2)^3)$$

$$\beta_f = t((\bar{b} - x + t/2)^4 - (x - t/2)^4)/2 + t\bar{a}^2((\bar{b} - x + t/2)^2 - (x - t/2)^2)/4$$

$$j = x_0 + (\beta_w + \beta_f)/(2I_y)$$

$$\bar{r}_0 = (x_0^2 + (I_x + I_y)/A)^{1/2}$$

$$H = \beta = 1 - (x_0/\bar{r}_0)^2$$

$$i_a = \sqrt{\frac{DI_y}{2S_x}}$$

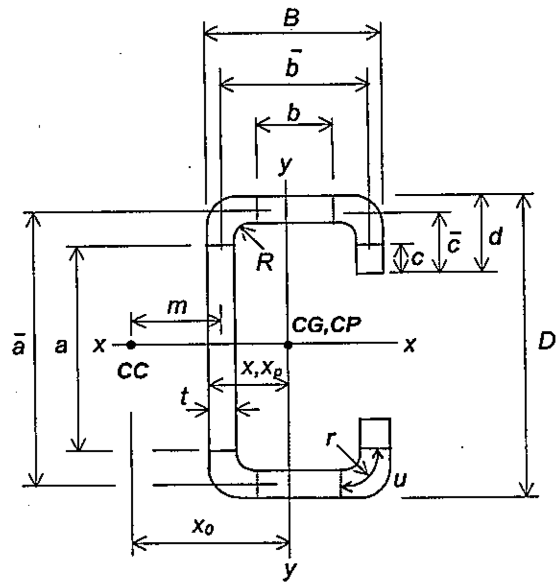
$$i_t = Bt/D$$

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}}$$

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2$$

E = módulo de elasticidad del acero (200000 MPa)

G = módulo de corte del acero (77200 MPa)



PERFIL CA PLEGADO

Parámetros auxiliares

$r = R + t/2$
$u = \pi r/2$
$a = D - 2(t + R)$
$\bar{a} = D - t$
$b = B - (t+R)2$
$\bar{b} = B - t/2$
$c = d - t - R$
$\bar{c} = d - t/2$

Área

$A = t(a + 2b + 2c + 4u)$

Centro de gravedad

$x = \{ B(A + t(D - 2d))/2 - t(D - 2d)(B - t/2) \} /A$

Centro plástico

se distinguen 3 casos :

caso 1 : si $t(b + c + u) \geq A/4 \rightarrow x_p \geq (R + t)$ (eje en el tramo recto del ala)

$x_p = (B + d - D/2)/2$

caso 2 : si $A_1 < (A/4 - at/2) \rightarrow t \leq x_p < (R + t)$ (eje en el codo) ; $A_1 = (r + t/2)^2 Arctan(t/r) - Rr/2$

$\theta_1 = (A/2 - at)/(2rt)$

$x_p = t/2 + r(1 - cos\theta_1)$

caso 3 : $x_p < t$ (eje en el alma)

en este caso, la ecuación trascendental que permite determinar x_p por algún método iterativo, es la siguiente :

$$\theta_2 = Arctan\left(\frac{\sqrt{2x_p(r+t/2)-x_p^2}}{r+t/2-x_p}\right)$$

$$ax_p + (r + t/2)^2(\theta_2 - \frac{1}{2} sen2\theta_2) - A/2 = 0$$

Momento de inercia

$I_x = 2t(0.0417a^3 + b(a/2 + r)^2 + 2u(a/2 + 0.637r)^2 + 0.298r^3 + 0.0833c^3 + c(a - c)^2/4)$

$I_y = 2t(0.0833b^3 + b(b/2 + r)^2 + 0.505r^3 + c(b + 2r)^2 + u(b + 1.637r)^2) - A(x - t/2)^2$

Módulo plástico

$Z_x = t(a^2/4 + \bar{a}b + \pi ra + 4r^2 + t^2/3 + ca - c^2)$

Caso 1 : $x_p \geq (R + t)$

$Z_y = t(a(x_p - t/2) + \pi rb + (b + r + t/2 - x_p)^2 + (x_p - r - t/2)^2 + 4r^2 + t^2/3 + 2c(B - x_p - t/2))$

Caso 2 : $t \leq x_p < (R + t)$

$Z_y = t\,[\,x_p\,(a + 3r(\theta_1 - \pi/2) - 2(b + c)) + r^2(2sen\theta_1 - 3\theta_1 + 2 - \pi/2) - rt(\pi/2 + \theta_1)/2 + B(b + \pi r + 2c) + t(t/6 - a/2 - c)\,]$

Caso 3 : $x_p < t$

$$Z_y = a(x_p^2 - tx_p + t^2/2) + (\tfrac{7}{8}x_p - r/2 - t/4)(r + t/2)^2(\theta_2 - \tfrac{1}{2}sen2\theta_2) + \tfrac{3}{2}\pi rt(r + t/2 - x_p + \tfrac{2}{3}b) + bt(B + 2x_p) +$$

$$+ 2ct(B - t/2 - x_p) + 2tr^2 + t^3/6$$

Propiedades flexo - torsionales

$$m = \bar{b}\left(\frac{3\bar{a}^2\bar{b} + \bar{c}\left(6\bar{a}^2 - 8\bar{c}^2\right)}{\bar{a}^3 + 6\bar{a}^2\bar{b} + \bar{c}\left(8\bar{c}^2 - 12\bar{a}\bar{c} + 6\bar{a}^2\right)}\right)$$

$J = t^3(a + 2b + 2c + 4u)/3$

$$C_w = \frac{t\bar{a}^2\bar{b}^2}{12}\,\frac{\left(2\bar{a}^3\bar{b} + 3\bar{a}^2\bar{b}^2 + 48\bar{c}^4 + 112\bar{b}\bar{c}^3 + 8\bar{a}\bar{c}^3 + 48\bar{a}\bar{b}\bar{c}^2 + 12\bar{a}^2\bar{c}^2 + 12\bar{a}^2\bar{b}\bar{c} + 6\bar{a}^3\bar{c}\right)}{\left(6\bar{a}^2\bar{b} + \left(\bar{a} + 2\bar{c}\right)^3 - 24\bar{a}\bar{c}^2\right)}$$

$x_o = x + m - t/2$ (distancia entre el centro de gravedad CG y el centro de corte CC)

$\beta_w = -(t\,\bar{a}^3(\,x - t/2)/12 + t\,\bar{a}(\,x - t/2)^3\,)$

$\beta_f = t((\bar{b} - x + t/2)^4 - (\,x - t/2)^4)/2 + t\bar{a}^2((\bar{b} - x + t/2)^2 - (x - t/2)^2)/4$

$\beta_l = 2\bar{c}t(\bar{b} - x + t/2)^3 + 2t(\bar{b} - x + t/2)((\bar{a}/2)^3 - (\bar{a}/2 - \bar{c})^3)/3$

$j = x_o + (\beta_w + \beta_f + \beta_l)/(2I_y)$

$\bar{r}_o = (x_o^2 + (I_x + I_y)/A)^{1/2}$

$H = \beta = 1 - (x_o\,/\,\bar{r}_o)^2$

$$i_a = \sqrt{\frac{DI_y}{2S_x}}$$

$i_t = Bt/D$

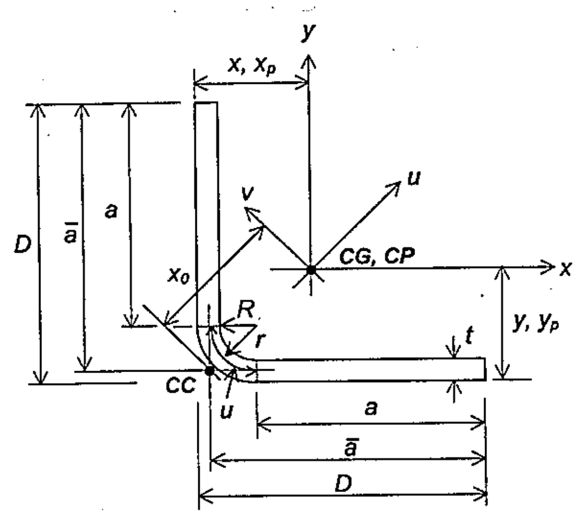
$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}}$$

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2$$

E = módulo de elasticidad del acero (200000 MPa)

G = módulo de corte del acero (77200 MPa)

PERIL L PLEGADO



Parámetros auxiliares

$r = R + t/2$
$u = \pi r/2$
$a = D - t - R$
$\bar{a} = D - t/2$

Área

$A = t(2a + u)$

Centro de gravedad

$x = y = t\{ a(r + t + a/2) + r ((r + t/2)\pi /2 - r) - t^2/12 \}/A$

Centro plástico

si $R \geq 1.2t$:

$x_p = y_p = t/2 + 0.2929r \geq t$

Momento de inercia

$I_x = I_y = (at^3 + a^3t)/12 + at\{(x - t/2)^2 + (D - x - a/2)^2\} + t\{ 0.1963r(4r^2 + t^2) - 0.1592(2r^2 + t^2/6)^2/r \} +$

$+ 1.5708rt(x - 0.3634r - t/2 + 0.0531t^2/r)^2$

$$I_{xy} = t\left[\frac{a}{2}(t-2x)(2R+2t-2x+a) + \frac{r}{8}\left(4r^2+t^2\right) + \frac{\left(\pi r\left(x-r-t/2\right)+2r^2+t^2/6\right)^2}{2\pi r} - \frac{\left(2r^2+t^2/6\right)^2}{2\pi r}\right]$$

$I_u = I_x - I_{xy}$

$I_v = I_x + I_{xy}$

Módulo plástico

$Z_x = Z_y = t(a(D - a/2 - t/2) + 0.4142r^2 - t^2/12)$

Propiedades flexo - torsionales

$J = t^3(2a + u)/3$

$$C_w = \frac{t^3\bar{a}^3}{18}$$

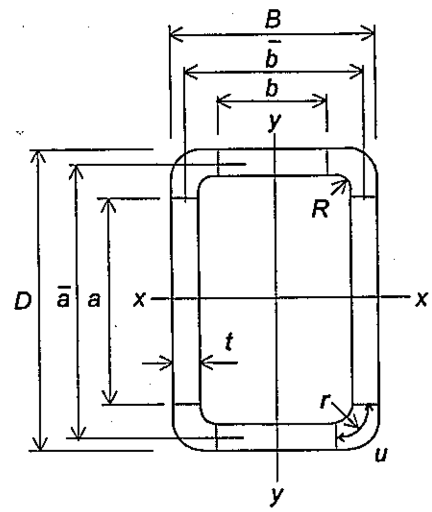
$x_0 = (x - t/2)\sqrt{2}$ (distancia entre el centro de gravedad CG y el centro de corte CC)

$j = \sqrt{2}\, t\, \bar{a}^4/(48I_v) + x_0$

$\bar{r}_0 = (x_0^2 + 2I_x /A)^{1/2}$

$H = \beta = 1 - (x_0 / \bar{r}_0)^2$

PERFIL CAJÓN PLEGADO



Parámetros auxiliares

$r = R + t/2$
$u = \pi r/2$
$a = D - 2(t + R)$
$\bar{a} = D - t$
$b = B - 2(t + R)$
$\bar{b} = B - t$

Área

$A = 2t(a + b + 2u)$

Momento de inercia

$$I_x = \frac{1}{6}\left(ta^3 + bt^3\right) + 2tb\left(\frac{a}{2} + r\right)^2 + \frac{t}{4\pi r}\left[(\pi r)^2\left(4r^2 + t^2\right) - 8\left(2r^2 + t^2/6\right)^2 + 2\left(\pi ra + 4r^2 + t^2/3\right)^2\right]$$

$$I_y = \frac{1}{6}\left(tb^3 + at^3\right) + 2ta\left(\frac{b}{2} + r\right)^2 + \frac{t}{4\pi r}\left[(\pi r)^2\left(4r^2 + t^2\right) - 8\left(2r^2 + t^2/6\right)^2 + 2\left(\pi rb + 4r^2 + t^2/3\right)^2\right]$$

Módulo plástico

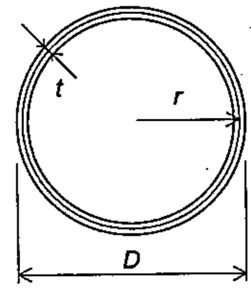
$$Z_x = \frac{ta^2}{2} + bt\left(a + 2r\right) + t\left(\pi ra + 4r^2 + t^2/3\right)$$

$$Z_y = \frac{tb^2}{2} + at\left(b + 2r\right) + t\left(\pi rb + 4r^2 + t^2/3\right)$$

Propiedades flexo - torsionales

$$J = \frac{2t\bar{a}^2\bar{b}^2}{\bar{a} + \bar{b}}$$

PERFIL TUBULAR CIRCULAR



Parámetros auxiliares

$D = D_{EXT}$
$D_{INT} = D - 2t$
$r = D/2 - t/2$

Área

$$A = \frac{\pi}{4}\left(D_{EXT}^2 - D_{INT}^2\right)$$

Momento de inercia

$$I = \frac{\pi}{64}\left(D_{EXT}^4 - D_{INT}^4\right)$$

Módulo plástico

$$Z = \frac{A}{\pi r}\left(2r^2 + \frac{t^2}{6}\right)$$

Propiedades flexo - torsionales

$J = 2I$

CODO DE 90^0

Parámetros auxiliares

$r = R + t/2$
$u = \pi r/2$

Área

$$A = \frac{\pi r t}{2} = ut$$

Centro de gravedad

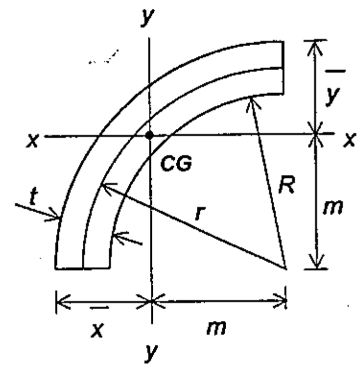
$$m = \frac{2r^2 + \frac{t^2}{6}}{\pi r}$$

$$\bar{x} = \bar{y} = r + \frac{t}{2} - m = \frac{\pi r\left(r + \frac{t}{2}\right) - 2r^2 - \frac{t^2}{6}}{\pi r}$$

Momento de inercia

$$I_x = I_y = \frac{t}{2}\left[\frac{\pi r}{8}\left(4r^2 + t^2\right) - \frac{\left(2r^2 + \frac{t^2}{6}\right)^2}{\pi r}\right]$$

$$I_{xy} = \frac{rt}{8}\left(4r^2 + t^2\right) - Am^2$$



FILETE CIRCULAR DE LAMINACIÓN

Área

$$A = R^2\left(1 - \frac{\pi}{4}\right)$$

Centro de gravedad

$$\bar{x} = \bar{y} = \left(\frac{10 - 3\pi}{12 - 3\pi}\right)R$$

Momento de inercia

$$I_x = I_y = \left(\frac{1}{3} - \frac{\pi}{16} - \frac{1}{36\left(1 - \frac{\pi}{4}\right)}\right)R^4$$

Producto de inercia

$$I_{xy} = \left(\frac{28 - 9\pi}{12 - 3\pi}\right)\frac{R^4}{24}$$